8.1 Find Angle Measures in Polygons

**THEOREM 8.1: POLYGON INTERIOR ANGLES THEOREM**

The sum of the measures of the interior angles of a convex \( n \)-gon is \((n-2) \cdot 180^\circ\).

Find the sum of the measures of the interior angles of the indicated convex polygon.

1. Decagon
   \[(10-2) \cdot 180 = 1440^\circ\]
2. 13-gon
   \[(13-2) \cdot 180 = 1980^\circ\]
3. 17-gon
4. 18-gon
5. 22-gon
6. 25-gon

The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.

1. \(1260^\circ\)
2. \(2160^\circ\)
3. \(3240^\circ\)

\[(n-2) \cdot 180 = 3240\]
\[n-2 = 18\]
\[n = 20 \text{ sides}\]

4. \(4680^\circ\)
5. \(5400^\circ\)

\[(n-2) \cdot 180 = 5400\]
\[n-2 = 30\]
\[n = 32 \text{ sides}\]
6. \(7560^\circ\)
Find the value of $x$.

1. $108^\circ \quad 72^\circ \quad 87^\circ \quad x^\circ = 360^\circ$
   
   \[ x + 72 + 108 + 87 = 360 \]
   \[ x = 93^\circ \]

2. $143^\circ \quad 77^\circ \quad 103^\circ \quad y^\circ = 360^\circ$
   
   \[ (7 - 2) \cdot 180 = 900 \]
   \[ 3x + 145 = 900 \]
   \[ 3x = 255 \]
   \[ x = 85^\circ \]

3. $156^\circ \quad 133^\circ \quad 99^\circ \quad 112^\circ \quad 145^\circ \quad 112^\circ \quad 112^\circ \quad 2x^\circ = 360^\circ$
   
   \[ 3x + 252 = 360 \]
   \[ 3x = 108 \]
   \[ x = 36^\circ \]

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**Theorem 8.2: Polygon Exterior Angles Theorem**

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is $360^\circ$.

Find the value of $x$.

1. $152^\circ \quad 123^\circ \quad x^\circ \quad 137^\circ$
   
   \[ x + 259 = 360 \]
   \[ x = 71^\circ \]

2. $67^\circ \quad 60^\circ \quad 59^\circ \quad x^\circ$
   
   \[ x + 308 = 360 \]
   \[ x = 52^\circ \]

3. $59^\circ \quad 69^\circ \quad 80^\circ \quad 64^\circ \quad x^\circ$
   
   \[ 3x + 252 = 360 \]
   \[ 3x = 108 \]
   \[ x = 36^\circ \]
Find the measures of an interior angle and an exterior angle of the indicated regular polygon.

1. Regular hexagon
   \[
   \text{Int\angle} : \frac{720^\circ}{6} = 120^\circ
   \]
   \[
   \text{Ext\angle} : \frac{360^\circ}{6} = 60^\circ
   \]

2. Regular decagon
   \[
   \text{Int\angle} : \frac{1440^\circ}{10} = 144^\circ
   \]
   \[
   \text{Ext\angle} : \frac{360^\circ}{10} = 36^\circ
   \]

3. Regular 15-gon

4. Regular 20-gon

5. Regular 30-gon

6. Regular 36-gon
   \[
   \text{Int\angle} : \frac{6120^\circ}{36} = 170^\circ
   \]
   \[
   \text{Ext\angle} : \frac{360^\circ}{36} = 10^\circ
   \]
find the value of \( n \) for each regular \( n \)-gon described.

1. Each interior angle of the regular \( n \)-gon has a measure of 108°.
\[
180 - 108 = 72° \quad n = \frac{360}{72} = 5 \text{ sides}
\]

2. Each interior angle of the regular \( n \)-gon has a measure of 135°.
\[
180 - 135 = 45° \quad \frac{360}{45} = 8 \text{ sides}
\]

3. Each interior angle of the regular \( n \)-gon has a measure of 144°.

4. Each exterior angle of the regular \( n \)-gon has a measure of 90°.
\[
n = \frac{360}{90} = 4 \text{ sides}
\]

5. Each exterior angle of the regular \( n \)-gon has a measure of 60°.

6. Each exterior angle of the regular \( n \)-gon has a measure of 40°.
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